## AQA

# LEVEL 3 MATHEMATICAL STUDIES (1350) 

## Specification

For teaching from September 2014 onwards
For exams in May/June 2016 onwards


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## Are you using the latest version of this specification?

- You will always find the most up-to-date version of this specification on our website at aqa.org.uk/1350
- We will write to you if there are significant changes to this specification.


## 1 Introduction

### 1.1 Why choose AQA for Level 3 Certificate Mathematical Studies

Maths is for everyone. It is diverse, engaging and essential in equipping students with the right skills to reach their future destination, whatever that may be. At AQA, we design qualifications and support to enable students to engage with, explore, enjoy and succeed in maths. By putting students at the heart of everything we do, our aim is to support teachers to shape what success in maths looks like for every student.

Our question papers are designed with students in mind. We're committed to ensuring that students are settled early in our exams and have the best possible opportunity to demonstrate their knowledge and understanding of maths, to ensure they achieve the results they deserve.

You can find out about all our Mathematics qualifications at aqa.org.uk/maths

### 1.2 Support and resources to help you teach

We know that support and resources are vital for your teaching and that you have limited time to find or develop good quality materials. So we've worked with experienced teachers to provide you with a range of resources that will help you confidently plan, teach and prepare for exams.

## Teaching resources

We have too many mathematical studies resources to list here so visit aqa.org.uk/1350 to see them all. They include:

- route maps to allow you to plan how to deliver the specification in the way that will best suit you and your students
- exemplar questions and mark schemes
- training courses to help you deliver AQA Mathematical Studies qualifications.


## Preparing for exams

Visit aqa.org.uk/1350 for everything you need to prepare for our exams, including:

- past papers, mark schemes and examiners' reports
- specimen papers and mark schemes for new courses
- exemplar student answers with examiner commentaries.


## Analyse your students' results with Enhanced Results Analysis (ERA)

Find out which questions were the most challenging, how the results compare to previous years and where your students need to improve. ERA, our free online results analysis tool, will help you see where to focus your teaching. Register at aqa.org.uk/era

For information about results, including maintaining standards over time, grade boundaries and our post-results services, visit aqa.org.uk/results

## Keep your skills up to date with professional development

Wherever you are in your career, there's always something new to learn. As well as subject-specific training, we offer a range of courses to help boost your skills.

- Improve your teaching skills in areas including differentiation, teaching literacy and meeting Ofsted requirements.
- Prepare for a new role with our leadership and management courses.

You can attend a course at venues around the country, in your school or online - whatever suits your needs and availability. Find out more at coursesandevents.aqa.org.uk

## Get help and support

Visit our website for information, guidance, support and resources at aqa.org.uk/1350

You can talk directly to the Mathematical Studies subject team

E: maths@aqa.org.uk
T: 01619573852

## 2 Specification at a glance

This qualification is linear. Linear means that students will sit all the exams at the end of their course.

## Subject content

## Compulsory content

3.1 Analysis of data (page 10)
3.2 Maths for personal finance (page 12)
3.3 Estimation (page 14)
3.4 Critical analysis of given data and models
(including spreadsheets and tabular) (page 14)

## Optional content

3.5 The normal distribution (page 15)
3.6 Probabilities and estimation (page 16)
3.7 Correlation and regression (page 16)
3.8 Critical path and risk analysis (page 18)
3.9 Expectation (page 18)
3.10 Cost benefit analysis (page 19)
3.11 Graphical methods (page 20)
3.12 Rates of change (page 20)
3.13 Exponential functions (page 21)

## Assessments

Paper 1

What's assessed:

- 3.1
- 3.2
- 3.3


## Assessed:

- written exam: 1 hour 30 minutes
- 60 marks
- scientific calculator or graphics calculator allowed (see section 5.9 for more information on calculators).


## Questions:

Copy of Preliminary material available in advance on eAQA and clean copy of Preliminary material to be provided in examination room.

Formulae sheet available.
No optional questions.

Paper 2A: Statistical techniques
What's assessed:

- 3.4
- 3.5
- 3.6
- 3.7

Students will be expected to draw on the mathematical content of Paper 1.

Students will be expected to develop and demonstrate confidence and competence in the understanding and application of mathematical modelling in the solution of problems related to the use of statistical techniques.

## Assessed:

- written exam: 1 hour 30 minutes
- 60 marks
- scientific calculator or graphics calculator allowed (see section 5.9 for more information on calculators).


## Questions:

Copy of Preliminary material available in advance on eAQA and clean copy of Preliminary material to be provided in examination room.

Formulae sheet available.
Statistical tables available.
No optional questions.

Or Paper 2B: Critical path and risk analysis

## What's assessed:

- 3.4
- 3.8
- 3.9
- 3.10

Students will be expected to draw on the mathematical content of Paper 1.

Students will be expected to develop and demonstrate confidence and competence in the understanding and application of mathematical modelling in the solution of problems related to decision making and the planning of projects.

## Assessed:

- written exam: 1 hour 30 minutes
- 60 marks
- scientific calculator or graphics calculator allowed (see section 5.9 for more information on calculators).


## Questions:

Copy of Preliminary material available in advance on eAQA and clean copy of Preliminary material to be provided in examination room.

Formulae sheet available.
No optional questions.

Or Paper 2C: Graphical techniques

## What's assessed:

- 3.4
- 3.11
- 3.12
- 3.13

Students will be expected to draw on the mathematical content of Paper 1.

Students will be expected to develop and demonstrate confidence and competence in the understanding and application of mathematical modelling in the solution of problems related to simple polynomial and exponential functions.

## Assessed:

- written exam: 1 hour 30 minutes
- 60 marks
- scientific calculator or graphics calculator allowed (see section 5.9 for more information on calculators).


## Questions:

Copy of Preliminary material available in advance on eAQA and clean copy of Preliminary material to be provided in examination room.

Formulae sheet available.
No optional questions.

## 3 Subject content

Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied. It is assumed that students will already have confidence and competence in the content presented in standard type within the GCSE Mathematics criteria. Students will make use of elements of this content when addressing problems within this Level 3 Certificate Mathematical Studies specification but this is not explicitly set out in subject content. This Level 3 Certificate Mathematical Studies specification aims to build on the knowledge, understanding and skills established in GCSE Mathematics.

Subject content that is expected to be used throughout this specification is listed below.

- Knowledge and use of the formula $y=m x+c$. It is also expected that students will be able to find the gradient of a straight line connecting two different points.
- It is expected that spreadsheets and tables will be used throughout the teaching of this Level 3 Certificate Mathematical Studies specification. Spreadsheet formulae will include:
- "=A1+A2+A3" to sum values in cells
- "=2*B3" to multiply a value in a given cell
- "=SUM(A1:A10)"
- Knowledge and use of the formulae for the circumference and the area of circle.
- Knowledge and use of the formulae for the perimeter of 2-D shapes, their areas and for calculating fractional areas of circles and composite shapes.
- Knowledge and use of the mathematical content of analysis of data and maths for personal finance elements.
- The ability to calculate surface areas of spheres, cones, pyramids and composite solids, including the application of the concepts of similarity including lengths in similar figures and Pythagoras' theorem applied to 2-D and 3-D figures.
- Students will make use of elements of the content from Paper 1 when addressing problems within each Paper 2.


### 3.1 Analysis of data

Students will be expected to develop and demonstrate confidence and competence in the understanding and application of statistical techniques, interpreting data and drawing conclusions in the solution of problems.

## D1 Data

\(\left.$$
\begin{array}{l|l|l}\text { D1.1 } & \text { Content } & \begin{array}{l}\text { appreciating the difference between } \\
\text { qualitative and quantitative data }\end{array} \\
\hline \text { D1.2 } & \begin{array}{l}\text { appreciating the difference between primary } \\
\text { and secondary data }\end{array} & \begin{array}{l}\text { including the difference between discrete } \\
\text { and continuous quantitative data }\end{array}
$$ <br>
\hline including the use of secondary data that <br>

have been processed eg grouped\end{array}\right]\)| D1.3 |
| :--- |
| primacting quantitative and qualitative |
| precondary data |

## D2 Collecting and sampling data

|  | Content | Additional information |
| :--- | :--- | :--- |
| D2.1 | inferring properties of populations or <br> distributions from a sample, whilst knowing <br> the limitations of sampling |  |
| D2.2 | appreciating the strengths and limitations <br> of random, cluster, stratified and quota <br> sampling methods and applying this <br> understanding when designing sampling <br> strategies | appreciating that improving accuracy by <br> removing bias and increasing sample size <br> may cost/save both time and money |

## D3 Representing data numerically

$\left.$| D3.1 | Content | calculating/identifying mean, median, mode, <br> quartiles, percentiles, range, interquartile <br> range, standard deviation |
| :--- | :--- | :--- | | either from raw data or from cumulative |
| :--- |
| frequency diagrams, stem-and-leaf |
| diagrams or box plots | \right\rvert\,

## D4 Representing data diagrammatically

| D4.1 | Content | constructing and interpreting diagrams for <br> grouped discrete data and continuous data, <br> knowing their appropriate use and reaching <br> conclusions based on these diagrams |
| :--- | :--- | :--- | | including histograms with equal and |
| :--- |
| unequal class intervals and cumulative |
| frequency graphs, box and whisker plots, |
| stem-and-leaf diagrams (including back-to- |
| back) |

### 3.2 Maths for personal finance

Students will be expected to develop and demonstrate confidence and competence in the understanding and application of the following calculations in the solution of problems relating to personal finance.

F1 Numerical calculations

|  | Content | Additional information |
| :--- | :--- | :--- |
| F1.1 | substituting numerical values into formulae, <br> spreadsheets and financial expressions | including bank accounts |
| F1.2 | using conventional notation for priority of <br> operations, including brackets, powers, <br> roots and reciprocals |  |
| F1.3 | applying and interpreting limits of accuracy, <br> specifying simple error intervals due to <br> truncation or rounding |  |
| F1.4 | finding approximate solutions to problems in <br> financial contexts |  |

F2 Percentages

| F2.1 | Content | Additional information |
| :--- | :--- | :--- |
| F2.2 | interpreting percentages and percentage <br> changes as a fraction or a decimal and <br> interpreting these multiplicatively |  |
| F2.3 | expressing one quantity as a percentage of <br> another | comparing two quantities using percentages |

## F3 Interest rates

|  | Content | Additional information |
| :--- | :--- | :--- |
| F3.1 | simple and compound interest | Annual Equivalent Rate (AER) |
| F3.2 | savings and investments |  |

## F4 Repayments and the cost of credit

|  | Content | Additional information |
| :--- | :--- | :--- |
| F4.1 | student loans and mortgages | Annual Percentage Rate (APR) |

## F5 Graphical representation

|  | Content | Additional information |
| :--- | :--- | :--- |
| F5.1 | graphical representation | plotting points to create graphs and <br> interpreting results from graphs in financial <br> contexts |

## F6 Taxation

|  | Content | Additional information |
| :--- | :--- | :--- |
| F6.1 | income tax, National Insurance, <br> Value Added Tax (VAT) |  |

## F7 Solution to financial problems

|  | Content | Additional information |
| :--- | :--- | :--- |
| F7.1 | the effect of inflation | Retail Price Index (RPI), Consumer Price <br> Index (CPI) |
| F7.2 | setting up, solving and interpreting the <br> solutions to financial problems, including <br> those that involve compound interest using <br> iterative methods |  |
| F7.3 | currency exchange rates including <br> commission |  |
| F7.4 | budgeting |  |

### 3.3 Estimation

Students should become familiar with and gain confidence in ideas concerning the formulation of mathematical models.

## E1 The modelling cycle

| E1.1 | Content | Additional information <br> representing a situation mathematically, <br> making assumptions and simplifications |
| :--- | :--- | :--- |
| E1.2 | students will engage in the tackling of <br> 'open' mathematical problem-solving where <br> there may not be a clear single approach or <br> 'correct' answer |  |
| selecting and using appropriate <br> mathematical techniques for problems and <br> situations |  |  |
| E1.4 | interpreting results in the context of a given <br> problem | evaluating methods and solutions including <br> how they may have been affected by <br> assumptions made |

E2 Fermi estimation

|  | Content | Additional information |
| :--- | :--- | :--- |
| E2.1 | making fast, rough estimates of quantities <br> which are either difficult or impossible to <br> measure directly |  |

In estimation, questions could draw upon all the content of Paper 1 and the content presented in standard type within GCSE Mathematics criteria.

### 3.4 Critical analysis of given data and models (including spreadsheets and tabular data)

Students will be expected to use the data and models they are given and to be mathematically critical of these.

C1 Presenting logical and reasoned arguments in context

|  | Content | Additional information |
| :--- | :--- | :--- |
| C1.1 | criticising the arguments of others |  |

C2 Communicating mathematical approaches and solutions

|  | Content | Additional information |
| :--- | :--- | :--- |
| C2.1 | summarising and report writing |  |

## C3 Analysing critically

| C3.1 | content <br> data | Additional information results from a model with real |
| :--- | :--- | :--- |
| C3.2 | critical analysis of data quoted in media, <br> political campaigns, marketing etc |  |

In addition to the content presented in standard type within GCSE Mathematics criteria, students will be expected to draw on the mathematical content of analysis of data and maths for personal finance.

In critical analysis, questions will concentrate on the analysis of numerical and graphical data.
Numerical data will usually be given in spreadsheet or tabular form.

### 3.5 The normal distribution

Students should be able to recognise that many things closely follow a normal distribution, eg heights of people, size of things produced by machines, errors in measurements, blood pressure, marks on a test etc. In these and similar situations, a graph of the distribution will have a 'bell' shaped curve.

S1 Properties of the normal distribution

| S1.1 | Content | Additional information |
| :--- | :--- | :--- |
| knowledge that this is a symmetrical <br> distribution and that the area underneath <br> the normal 'bell' shaped curve represents <br> probability | knowledge that approximately $\frac{2}{3}$ of <br> observations lie within 1 standard deviation <br> of the mean and that approximately 95\% of <br> observations lie within 2 standard deviations <br> of the mean |  |

## S2 Notation

| S2.1 | Content | Additional information |
| :--- | :--- | :--- |
| use of the notation $\mathrm{N}\left(\mu, \sigma^{2}\right)$ to describe a <br> normal distribution in terms of mean and <br> standard deviation | use of the notation $\mathrm{N}(0,1)$ for the <br> standardised normal distribution with <br> mean = 0 and standard deviation =1 |  |

## S3 Calculating probabilities

| S3.1 | Content | Additional information |
| :--- | :--- | :--- |
| using a calculator or tables to find <br> probabilities for normally distributed data <br> with known mean and standard deviation | the finding of an unknown mean or standard <br> deviation by making use of percentage <br> points will not be required |  |

### 3.6 Probabilities and estimation

S4 Population and sample

|  | Content | Additional information |
| :--- | :--- | :--- |
| S4.1 | understanding what is meant by the term <br> 'population' in statistical terms |  |
| S4.2 | developing ideas of sampling to include the <br> concept of a simple random sample from a <br> population |  |

## S5 The mean of sample size $n$

| S5.1 | Content | Additional information <br> called a a 'point estimate' for the mean of the <br> population |
| :--- | :--- | :--- | | appreciating that accuracy is likely to be |
| :--- |
| improved by increasing the sample size |

## S6 Confidence intervals

| S6.1 | Content | Additional information |
| :--- | :--- | :--- |
| confidence intervals for the mean of a <br> normally distributed population of known <br> variance using $\frac{\sigma^{2}}{n}$ | confidence intervals will always be <br> symmetrical <br> the confidence level required and the <br> sample size will always be stated |  |

### 3.7 Correlation and regression

## S7 Correlation

| S7.1 | Content | Additional information |
| :--- | :--- | :--- |
| recognising when pairs of data are |  |  |
| uncorrelated, correlated, strongly correlated, |  |  |
| positively correlated and negatively correlated |  |  |$\quad$ | S7.2 | appreciating that correlation does not <br> necessarily imply causation | understanding the idea of an outlier |
| :--- | :--- | :--- | | identifying and understanding outliers and |
| :--- |
| make decisions whether or not to include |
| them when drawing a line of best fit |

## S8 The product moment correlation coefficient (pmcc)

| S8.1 | Content | Additional information |
| :--- | :--- | :--- |
| understanding that the strength of |  |  |
| correlation is given by the pmcc |  |  |$\quad$ | S8.2 | understanding that pmcc always has a value <br> in the range from -1 to +1 |
| :--- | :--- |
| S8.3 | appreciating the significance of a positive, <br> zero or negative value of pmcc in terms of <br> correlation of data |

## S9 Regression lines

| S9.1 | Content | the plotting of data pairs on scatter <br> diagrams and the drawing, by eye, of a line <br> of best fit through the mean point |
| :--- | :--- | :--- | the idea of residuals will not be required $\quad$ (line | S9.2 | understanding the concept of a regression <br> line |
| :--- | :--- |
| S9.3 | plotting a regression line from its equation |
| S9.4 | using interpolation with regression lines to <br> make predictions |
| S9.5 | understanding the potential problems of <br> extrapolation |

S10 Calculations

| S10.1 | Content | where raw data is given, students will be <br> expected to use a calculator to calculate the <br> pmcc and the equation of the regression line |
| :--- | :--- | :--- | | calculations from grouped data will not be |
| :--- |
| required |

### 3.8 Critical path analysis

## R1 Compound projects

|  | Content | Additional information |
| :--- | :--- | :--- |
| R1.1 | representing compound projects by activity <br> networks |  |
| R1.2 | activity-on-node representation will be used |  |

## R2 Critical activities

| R2.1 | Content | Additional information |
| :--- | :--- | :--- |
|  | using early time and late time algorithms to <br> identify critical activities and find the critical <br> path(s) |  |

R3 Gantt charts

|  | Content | Additional information |
| :--- | :--- | :--- |
| R3.1 | using Gantt charts (cascade diagrams) to <br> present project activities |  |

### 3.9 Expectation

## R4 Probability

| R4.1 | Content | Additional information |
| :--- | :--- | :--- |
| understanding that uncertain outcomes <br> can be modelled as random events with <br> estimated probabilities | knowing that the probabilities of an <br> exhaustive set of outcomes sum to one |  |
| R4.2 | applying ideas of randomness, fairness and <br> equally likely events to calculate expected <br> outcomes |  |

R5 Diagrammatic representations

|  | Content | Additional information |
| :--- | :--- | :--- |
| R5.1 | understanding and applying Venn diagrams <br> and simple tree diagrams | - understanding that $\mathrm{P}(A)$ means the <br> probability of event $A$ <br> understanding that $\mathrm{P}\left(A^{\prime}\right)$ means the <br> probability of not event $A$ |
|  | understanding that $\mathrm{P}(A \cup B)$ means the <br> probability of event $A$ or $B$ or both <br> understanding that $\mathrm{P}(A \cap B)$ means the <br> probability of event $A$ and $B$ |  |

## R6 Combined events

| R6.1 | Content | Additional information |
| :--- | :--- | :--- |
|  | calculating the probability of combined <br> events: <br> - both $A$ and $B ;$ <br> - either $A$ nor $B ;$ | to include independent and dependent <br> events |

## R7 Expected value

|  | Content | Additional information |
| :--- | :--- | :--- |
| R7.1 | calculating the expected value of quantities <br> such as financial loss or gain |  |

### 3.10 Cost benefit analysis

## R8 Living with uncertainty

| R8.1 | Content | Additional information |
| :--- | :--- | :--- |
|  | understanding that many decisions have <br> to be made when outcomes cannot be <br> predicted with certainty |  |

R9 Control measures

| R9.1 | Content | Additional information <br> understanding that the actions that can be <br> taken to reduce or prevent specific risks <br> may have their own costs |
| :--- | :--- | :--- | | including the costs and benefits of |
| :--- |
| insurance |

R10 Risk analysis

| R10.1 | using probabilities to calculate expected <br> values of costs and benefits of decisions | other factors must be considered, for <br> example: <br> - the regulatory framework (eg compulsory <br> insurance) <br> - minimising the maximum possible loss |
| :--- | :--- | :--- |
| R10.2 | understanding that calculating an expected <br> value is an important part of such decision <br> making |  |

### 3.11 Graphical methods

G1 Graphs of functions

| G1.1 | Content | sketching and plotting curves defined by <br> simple equations |
| :--- | :--- | :--- | | Additional information |
| :--- |
| knowing the shapes of the graphs of linear, |
| quadrac, cubic and exponential functions |
| will be expected |

G2 Intersection points

$\left.$| G2.1 | Content | Additional information |
| :--- | :--- | :--- |
| plotting and interpreting graphs (including |  |  |
| exponential graphs) in real contexts, to find |  |  |
| approximate solutions to problems |  |  |$\quad$| including understanding the potential |
| :--- |
| problems of extrapolation | \right\rvert\, | G2.2 | interpreting the solutions of equations as the <br> intersection points of graphs and vice versa |
| :--- | :--- |

### 3.12 Rates of change

G3 Gradient
$\left.\begin{array}{l|l|l}\text { G3.1 } & \text { Content } & \text { Additional information } \\ \hline \text { interpreting the gradient of a straight line } & \\ \hline \text { graph as a rate of change }\end{array} \quad \begin{array}{l}\text { interpreting the gradient at a point on a } \\ \text { curve as an instantaneous rate of change }\end{array} \quad \begin{array}{l}\text { understanding that some maximum and } \\ \text { minimum points on curves occur where the } \\ \text { gradient is zero }\end{array}\right]$

## G4 Average speed

| G4.1 | Content | Additional information |
| :--- | :--- | :--- |
| knowing that the average speed of an <br> object during a particular period of time is <br> given by $\frac{\text { distance travelled }}{\text { time taken }}$ |  |  |

G5 Speed and acceleration

| G5.1 | Knowing that the gradient of a <br> distance-time graph represents speed and <br> that the gradient of a velocity-time graph <br> represents acceleration | Additional information |
| :--- | :--- | :--- |

### 3.13 Exponential functions

## G6 The function $a^{x}$

| G6.1 | Content | Additional information |
| :--- | :--- | :--- |
| using a calculator to find values of such a | the laws of logarithms will not be required |  |
| function |  |  | | using a calculator log function to solve |
| :--- |
| equations of the form $a^{x}=b$ and $\mathrm{e}^{k x}=b$ |$\quad$

G7 The number e

| G7.1 | Content | understanding that e has been chosen as <br> the standard base for exponential functions |
| :--- | :--- | :--- |
| Aditional information <br> the graph that the gradient at any point on $y=\mathrm{e}^{x}$ is equal to the $y$ value of <br> that point |  |  |

## G8 Exponential growth and decay

|  | Content | Additional information |
| :--- | :--- | :--- |
| G8.1 | formulating and using equations of the form <br> $y=C a^{x}$ and $y=C \mathrm{e}^{k x}$ |  |
| G8.2 | using exponential functions to model growth <br> and decay in various contexts |  |

## 4 Scheme of assessment

Find past papers and mark schemes, and specimen papers for new courses, on our website at aqa.org.uk/pastpapers

This specification is designed to be taken over two years with all assessments taken at the end of the course.

Level 3 Certificate Mathematical Studies exams and certification for this specification are available for the first time in May/June 2016 and then every May/June for the life of the specification.

This is a linear qualification. In order to achieve the award, students must complete all exams in May/ June in a single year. All assessments must be taken in the same series.

All exams in mathematics must include questions that allow students to draw on elements from within and across different topic areas, and questions that allow students to provide extended responses.

All materials are available in English only.

### 4.1 Aims

This Level 3 Certificate Mathematical Studies qualification will consolidate students' mathematical understanding, build their confidence and competence in applying mathematical techniques to solve a range of problems and introduce them to new techniques and concepts that will prepare them for further study and future employment within a broad range of academic, professional and technical fields.

Mathematical Studies aims to prepare students for the mathematical demands of higher education and work where there is a distinct mathematical or statistical element, but where the mathematical demands do not stretch to a requirement for A-level Mathematics.

A course of study leading to this qualification should enable students to:

- study a mathematics curriculum that is integrated with other areas of their study, work or interest leading to the application of mathematics in these areas
- develop mathematical modelling, evaluating and reasoning skills
- solve problems some of which will not be well defined and may not have a unique solution
- solve substantial and real life problems encountered by adults
- use ICT as an exploratory tool for developing mathematical understanding and when solving problems
- develop skills in the communication, selection, use and interpretation of their mathematics
- enjoy mathematics and develop confidence in using mathematics.


### 4.2 Assessment objectives

The assessment objectives (AOs) have been set by AQA under guidance from the Department for Education and Ofqual.

The exams will measure how students have achieved the following assessment objectives.

## A01: Use and apply standard techniques

Students should be able to:

- accurately recall facts, terminology and definitions
- accurately carry out set tasks in contexts requiring single- and multi-step solutions.


## AO2: Select appropriate techniques to solve problems in a mathematical or non-mathematical context and analyse data and represent situations mathematically

Students should be able to:

- select the mathematics required to solve a problem
- analyse data and/or given information in order to draw conclusions
- represent situations mathematically.


## AO3: Devise strategies to solve problems where the method is not obvious and communicate processes and results

Students should be able to:

- devise strategies to solve problems where the method is not obvious
- make realistic assumptions to enable solutions to be obtained
- evaluate the reasonableness of a solution
- communicate clearly using appropriate mathematical notation, language and diagrammatic representation.

The balance of the assessment objectives may not be equal on each paper.
Weighting of assessment objectives for Level 3 Certificate Mathematical Studies

| Assessment objectives (AOs) | Component weightings (approx \%) |  | Overall weighting <br> (approx \%) |
| :--- | :--- | :--- | :--- |
|  | Paper 1 | Paper 2 | $25-30$ |
| AO1 | $25-30$ | $25-30$ | $31-40$ |
| AO2 | $31-40$ | $31-40$ | $31-40$ |
| AO3 | $31-40$ | $31-40$ | 100 |
| Overall weighting of components | 50 | 50 |  |

### 4.3 Assessment weightings

The marks awarded on the papers will be scaled to meet the weighting of the components. Students' final marks will be calculated by adding together the scaled marks for each component. Grade boundaries will be set using this total scaled mark. The scaling and total scaled marks are shown in the table below.

| Component | Maximum <br> raw mark | Scaling factor | Maximum <br> scaled mark |
| :--- | :--- | :--- | :--- |
| Paper 1 | 60 | x 1 | 60 |
| Paper 2A (Statistical techniques) or <br> Paper 2B (Critical path and risk analysis) or <br> Paper 2C (Graphical techniques) | 60 | x 1 | 60 |
|  | Total scaled mark: | 120 |  |

## 5 General administration

You can find information about all aspects of administration, as well as all the forms you need, at aqa.org.uk/examsadmin

### 5.1 Entries and codes

You only need to make one entry for each qualification - this will cover all the question papers and certification.

Every specification is given a national discount (classification) code by the Department for Education (DfE), which indicates its subject area.

If a student takes two specifications with the same discount code, Further and Higher Education providers are likely to take the view that they have only achieved one of the two qualifications. Please check this before your students start their course.

| Qualification title | Option | AQA entry <br> code | DfE discount <br> code |
| :--- | :--- | :--- | :--- |
| AQA Level 3 Certificate <br> Mathematical Studies | with statistical techniques | 1350 A | 2350 |
| AQA Level 3 Certificate <br> Mathematical Studies | with critical path and risk analysis | 1350 B | 2350 |
| AQA Level 3 Certificate <br> Mathematical Studies | with graphical techniques | 1350 C | 2350 |

This specification complies with Ofqual's General Conditions of Recognition and all relevant regulatory documents.

The Ofqual qualification accreditation number (QAN) is 601/4945/0

### 5.2 Overlaps with other qualifications

There is some overlap between this specification and AQA's GCSE Mathematics, AQA's GCSE Statistics and with AQA's Functional Skills qualifications in Mathematics at Level 1 and Level 2. Some overlap also exists with this specification and AQA's Level 2 Certificate in Further Mathematics and with AQA's suite of Free-Standing Mathematics Qualifications (FSMQs).

### 5.3 Awarding grades and reporting results

The Level 3 Certificate qualification will be graded on a five-point scale: A, B, C, D and E.
Students who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

### 5.4 Re-sits and shelf life

Students can re-sit the qualification as many times as they wish, within the shelf life of the qualification.

### 5.5 Previous learning and prerequisites

Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied.

It is expected that students engaging on a course of study leading to this qualification will have already achieved a 'good' pass (currently grade C or above) in GCSE Mathematics or an equivalent Level $1 / 2$ qualification.

Hence, it is assumed that all students will have competence and confidence in the content presented in standard type in the Department for Education's GCSE mathematics content and assessment objectives document. This is the content listed as basic foundation content in the AQA GCSE Mathematics specification (8300) for exams from 2017 onwards. Students will make use of this content when addressing problems within the assessment of this specification but this content is not explicitly set out in this specification. All other content required for this specification is listed in Section 3 of this specification.

### 5.6 Access to assessment: diversity and inclusion

General qualifications are designed to prepare students for a wide range of occupations and further study. Therefore our qualifications must assess a wide range of competences.

The subject criteria have been assessed to see if any of the skills or knowledge required present any possible difficulty to any students, whatever their ethnic background, religion, sex, age, disability or sexuality. If any difficulties were encountered, the criteria were reviewed again to make sure that tests of specific competences were only included if they were important to the subject.

As members of the Joint Council for Qualifications (JCQ) we participate in the production of the JCQ document Access Arrangements and Reasonable Adjustments: General and Vocational qualifications. We follow these guidelines when assessing the needs of individual students who may require an access arrangement or reasonable adjustment. This document is published on the JCQ website at jcq.org.uk

## Students with disabilities and special needs

We can make arrangements for disabled students and students with special needs to help them access the assessments, as long as the competences being tested are not changed. Access arrangements must be agreed before the assessment. For example, a Braille paper would be a reasonable adjustment for a Braille reader but not for a student who does not read Braille.

We are required by the Equality Act 2010 to make reasonable adjustments to remove or lessen any disadvantage that affects a disabled student.

If you have students who need access arrangements or reasonable adjustments, you can apply using the Access arrangements online service at aqa.org.uk/eaqa

## Special consideration

We can give special consideration to students who have been disadvantaged at the time of the assessment through no fault of their own - for example a temporary illness, injury or serious problem such as the death of a relative. We can only do this after the assessment.

Your exams officer should apply online for special consideration at aqa.org.uk/eaqa
For more information and advice about access arrangements, reasonable adjustments and special consideration please see aqa.org.uk/access or email accessarrangementsqueries@aqa.org.uk

### 5.7 Working with AQA for the first time

If your school or college has not previously offered any AQA specification, you need to register as an AQA centre to offer our specifications to your students. Find out how at aqa.org.uk/becomeacentre

If your school or college is new to this specification, please let us know by completing an Intention to enter form. The easiest way to do this is via e-AQA at aqa.org.uk/eaqa

### 5.8 Private candidates

A private candidate is someone who enters for exams through an AQA-approved school or college but is not enrolled as a student there.

If you are a private candidate you may be self-taught, home-schooled or have private tuition, either with a tutor or through a distance learning organisation. You must be based in the UK.

If you have any queries as a private candidate, you can:

- speak to the exams officer at the school or college where you intend to take your exams
- visit our website at aqa.org.uk/examsadmin
- email: privatecandidates@aqa.org.uk


### 5.9 Materials for use in the examination

For all question papers, students are expected to write in black pen and have mathematical instruments available for use in the exam. These instruments are defined as:

- pencil (for use in diagrams only - no colour pencils)
- ruler
- pair of compasses
- protractor.

Calculators are allowed in all papers in this specification. The rules are the same as for any other examination - please see the JCQ document Instructions for Conducting Examinations for further details.

Most models of scientific or graphics calculator are allowed. However, calculators that feature a 'Computer Algebra System' (CAS) are not allowed. It is usually clear from the manufacturer's specifications whether a model has this feature.

It is expected that students are familiar with the use of a scientific calculator or a graphics calculator. They should also use a scientific calculator when studying this specification and sitting its exams.

Preliminary materials will be available in advance of the examination via eAQA, for issue from 1 March each year. A clean copy of the Preliminary material will be provided as an insert to the examination paper. Students must not take a copy of their pre-release Preliminary material into the examination room.

A formulae sheet will be provided for use with all mathematical studies exams.
A statistical tables sheet will be provided for use with Paper 2A only.

## 6 Grade descriptions

These descriptions give a general indication of the level of attainment characteristic of each grade. These descriptions should be interpreted in relation to the content outlined in the Subject content section of this specification; they are not designed to define that content.

The grade awarded will depend on how well the student has met the assessment objectives (see the Scheme of assessment section). If a student has performed less well in some areas, this may be balanced by better performances in others.

## Level 3 Certificate Mathematical Studies

## Grade A

Within the context of Level 3 Certificate Mathematical Studies, students demonstrate a good understanding and knowledge of the mathematical facts, concepts and techniques that are needed, drawing on the full range of defined and assumed content to carry out set tasks successfully.

Students manipulate mathematical expressions and use graphs, sketches, tables and diagrams, all with high accuracy and skill. They use mathematical language and symbols correctly and effectively in representing situations mathematically. When confronted with unstructured problems, they can often devise and implement an effective solution strategy, communicating it appropriately and effectively. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Students recall or recognise almost all the standard models and techniques that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretation of their results in the context of the original situation. Their responses include mathematical justifications, explaining their solutions to problems involving a number of features or variables. They make intelligent comments on the modelling assumptions and suggest possible refinements to the model.

Students understand how almost all situations presented in the examination may be translated into mathematics. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical requirements from given data or other mathematical information.

Students make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy without prompting.

## Grade E

Within the context of Level 3 Certificate Mathematical Studies, students demonstrate some understanding and knowledge of the mathematical facts, concepts and techniques required, showing competence and confidence in applying assumed content and some technical fluency with the defined content.

Students manipulate mathematical expressions and use graphs, sketches, tables and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly to represent situations and the processes they use in working through well-defined problems can be followed.

Students recall or recognise some of the standard models and techniques that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original situation, but make little comment on possible refinements to a model.

Students sometimes understand how situations presented in the examination may be translated into mathematics. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical requirements from given data or other mathematical information.

Students often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They sometimes present results to an appropriate degree of accuracy without prompting.

## 7 Formulae sheet

These formulae are not required to be learnt. A clean copy of this formulae sheet will be issued to you in the examination.

### 7.1 Volume and surface area

| Shape | Volume | Surface area |
| :--- | :--- | :--- |
| Cone | $V=\frac{1}{3} \pi r^{2} h$ | $A=\pi r l+\pi r^{2}$ |
| Sphere | $V=\frac{4}{3} \pi r^{3}$ | $A=4 \pi r^{2}$ |
| Pyramid | $V=\frac{1}{3}$ base $\times h$ |  |

### 7.2 Financial calculation - Annual Equivalent Rate (AER)

The annual equivalent interest rate (AER), $r$, is given by
$r=\left(1+\frac{i}{n}\right)^{n}-1$
where $i$ is the nominal interest rate, and $n$ the number of compounding periods per year.
Note: the values of $i$ and $r$ should be expressed as decimals.

### 7.3 Financial calculation - Annual Percentage Rate (APR)

The annual percentage interest rate (APR) is given by
$C=\sum_{k=1}^{m}\left(\frac{A_{k}}{(1+i)^{k}}\right)$
where $£ C$ is the amount of the loan, $m$ is the number of repayments, $i$ is the APR expressed as a decimal, $£ A_{k}$ is the amount of the $k$ th repayment, $t_{k}$ is the interval in years between the start of the loan and the $k$ th repayment.

It may be assumed that there are no arrangement or exit fees.

## 8 Statistical tables

These statistical tables are required for use in Paper 2A only.

### 8.1 Table 1 - normal distribution function

The table gives the probability, $p$, that a normally distributed random variable $Z$, with mean $=0$ and variance $=1$, is less than or equal to $z$.


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| $\stackrel{5}{\circ}$ | $\begin{aligned} & \text { O} \\ & \text { O} \\ & \text { O} \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \substack{0 \\ 0 \\ 0} \end{aligned}$ | $\begin{aligned} & \stackrel{N}{N} \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & N \\ & \underset{N}{N} \\ & \dot{O} \end{aligned}$ | $\begin{aligned} & \text { 웅 } \\ & \stackrel{0}{0} \\ & \mathbf{O} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\circ} \\ & \underset{O}{0} \\ & \dot{O} \end{aligned}$ | $\begin{aligned} & \hat{O} \\ & \underset{\sim}{N} \\ & \underset{O}{2} \end{aligned}$ | $\begin{aligned} & \frac{1}{\Gamma} \\ & \vdots \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & 0 \\ & 0 \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \hline 0 \\ & 0 . \\ & 00 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ |  |  | $$ | 0 <br> 0 <br> 0 <br> 0 <br> 0 | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \hline 0 \end{aligned}$ |  | $\begin{aligned} & \text { M} \\ & \underset{\sim}{\lambda} \\ & \underset{0}{2} \end{aligned}$ |  |
| $8$ | $\begin{aligned} & \text { O} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\infty$ 0 0 0 0 | $\begin{aligned} & 0 \\ & \\ & \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \overline{2} \\ & \stackrel{\rightharpoonup}{0} \\ & \vdots \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \stackrel{H}{0} \\ & \underset{0}{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{6} \\ & \vdots \\ & 0 \end{aligned}$ | $$ |  | $\begin{aligned} & \underset{r}{t} \\ & \underset{\infty}{\infty} \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & \text { O } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{W}{M} \\ & \underset{\sim}{\infty} \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { M } \\ & \substack{0 \\ 0 \\ 0 \\ 0 \\ \hline} \end{aligned}$ | $\begin{aligned} & \infty \\ & \hline \\ & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\mathrm{O}} \\ & \stackrel{0}{\mathrm{O}} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \dot{J} \\ & \underset{\sim}{\sigma} \\ & \vdots \\ & \hline \end{aligned}$ | の ल O 0 | $\begin{aligned} & 0 \\ & \stackrel{N}{0} \\ & \underset{\sim}{0} \end{aligned}$ | $$ |  | $\begin{aligned} & \infty \\ & \stackrel{\infty}{N} \\ & \underset{\sim}{0} \\ & \hline \end{aligned}$ | $\stackrel{\text { N}}{\stackrel{N}{N}}$ |
|  | $\bigcirc$ | $\stackrel{\square}{0}$ | $\stackrel{y}{0}$ | $\stackrel{m}{0}$ | $\stackrel{\square}{\circ}$ | $\xrightarrow[0]{0}$ | $\stackrel{\ominus}{\circ}$ | N | ${ }_{0}^{\infty}$ | $8$ | $\stackrel{O}{r}$ | ᄃ. | ̣ | అ. | $\underset{\sim}{\underset{\sim}{*}}$ | م | $\underline{0}$ | $\bigcirc$ | $\stackrel{\infty}{\Gamma}$ | $\bigcirc$ | 0 |


| N | $\stackrel{\overline{\mathrm{N}}}{ }$ | $\stackrel{N}{N}$ | $\stackrel{m}{\mathrm{~N}}$ | $\stackrel{\rightharpoonup}{\sim}$ | $\begin{aligned} & \text { م̣ } \end{aligned}$ | $\stackrel{\bullet}{N}$ | $\stackrel{N}{N}$ | $\begin{aligned} & \infty \\ & \stackrel{N}{N} \end{aligned}$ | $\stackrel{\circ}{\mathrm{N}}$ | $\stackrel{0}{\circ}$ | $\stackrel{\Gamma}{\text { ¢ }}$ | $\stackrel{N}{\mathrm{~m}}$ | $\stackrel{m}{m}$ | $\stackrel{+}{\stackrel{+}{\circ}}$ | $\stackrel{\circ}{\mathrm{M}}$ | $\stackrel{\odot}{\circ}$ | $\stackrel{N}{\mathrm{~m}}$ | $\stackrel{\infty}{\infty}$ | $\stackrel{\square}{\text { ® }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\pm$ $N$ 0 0 0 | O <br> 0 <br> $\infty$ <br> 0 <br> 0 <br> 0 | $\infty$ 0 0 0 0 0 | $\overline{0}$ $\underset{O}{0}$ 0 0 |  |  | 0 $\stackrel{0}{1}$ $\stackrel{3}{8}$ 0 | N 0 0 0 0 0 | $\overline{0}$ 0 0 0 0 | 8 <br> 8 <br> 8 <br> 8 | ö N － － － | $\circ$ <br> 0 <br> 0 <br> 8 <br> 0 | 10 <br> 0 <br> 8 <br> 8 <br> 0 <br> 0 | $\circ$ $\stackrel{\circ}{\circ}$ g 0 0 |  |  |  | 10 <br> 8 <br> 8 <br> 8 <br> 0 |  |
| $\begin{aligned} & \infty \\ & \hline- \\ & \hline- \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { N } \\ & \text { O } \\ & 0 \\ & 0 \end{aligned}$ | O <br> $\infty$ <br> $\infty$ <br> 0 <br> 0 <br> 0 |  | $\stackrel{3}{7}$ $\underset{0}{3}$ 0 | 0 0 0 0 0 0 0 | N © O－ 0 0 | $\stackrel{\infty}{N}$ $\stackrel{3}{\circ}$ 0 0 | $\bar{\circ}$ 0 0 0 0 0 | 0 0 0 0 0 0 | 0 0 0 0 0 0 |  | $\stackrel{\infty}{\square}$ <br> $\stackrel{8}{8}$ <br> - <br> - |  |  |  | $\infty$ <br> 0 <br> 0 <br> 8 <br> 0 <br> 0 |  | $\circ$ <br> 8 <br> 8 <br> 8 <br> 8 | N <br> － <br> － <br> － <br> - |
| O | 8 0 0 0 0 0 | O 0 $\infty$ 0 0 0 | 「 <br> 「 <br> 万 | $\underset{\sim}{\sim}$ $\underset{\sim}{O}$ $\underset{O}{\circ}$ | $\stackrel{1}{8}$ $\boxed{8}$ 0 0 |  |  |  | 5 0 0 0 0 | 0 0 0 0 0 0 |  | $\circ$ <br> $\stackrel{+}{\square}$ <br> $\stackrel{8}{\circ}$ <br> - |  |  | N O O o 0 | $\infty$ 0 0 0 0 0 0 |  | 10 <br> 8 <br> 8 <br> 8 <br> 0 | $\circ$ <br> ® <br> － <br> － <br> - |
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| $\begin{aligned} & 10 \\ & 0 \end{aligned}$ | $\begin{aligned} & \mathbb{N} \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{0} \\ & 0 \end{aligned}$ | $\stackrel{\infty}{\sim}$ | $\overline{6}$ <br> 8 <br> 8 <br> 8 | 0 0 $\underset{\sim}{0}$ 0 0 0 |  | $\infty$ 0 0 0 0 0 0 |  | $\begin{aligned} & \Gamma_{0} \\ & \stackrel{1}{\circ} \\ & \underset{o}{\circ} \end{aligned}$ | $\square$ <br> -8 <br> 0 <br> 0 <br> 0 | 0 0 0 0 0 0 0 | $\stackrel{\infty}{\circ}$ |  | $\circ$ <br> 8 <br> 8 <br> 8 <br> 0 | N <br> $\stackrel{1}{2}$ <br> O <br> 0 | － © O 0 0 | $$ |  <br> ন <br> － <br> － <br> - |  | $\circ$ <br> O <br> － <br> － <br> - |
| O | $\begin{aligned} & N \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { Lo } \\ & \underset{\sim}{\infty} \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ |  |  | $\circ$ <br> + <br> -8 <br> - | $\begin{aligned} & 10 \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | 0 <br> 0 <br> 0 <br> - <br> 0 | $\begin{aligned} & \text { N } \\ & \underset{\sim}{\circ} \\ & 0 \end{aligned}$ | 0 0 0 0 0 0 | $\begin{aligned} & \mathbb{O} \\ & \infty \\ & \infty \\ & \hline 0 \\ & 0 \\ & \hline 0 \end{aligned}$ | $\circ$ <br> $\stackrel{\circ}{\sigma}$ <br> － <br> - | $\begin{aligned} & \text { 아 } \\ & \text { O } \\ & \text { or } \\ & \text { ó } \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{0}{0} \\ & \text { O} \\ & \text { O} \\ & \hline 0 \end{aligned}$ | ন <br> － <br> － <br> － <br> - | $\begin{aligned} & \circ \\ & 0 \\ & \circ \\ & \hline 8 \\ & \hline 0 \end{aligned}$ | 0 <br> 0 <br> 0 <br> 8 <br> 0 <br> 0 |  <br> － <br> ® <br> 0 | $\square$ <br> 8 <br> - <br> - | $\circ$ <br> 8 <br> 8 <br> 0 <br> 0 |
| $\begin{aligned} & \mathrm{m} \\ & \hline \end{aligned}$ | $\overline{7}$ <br> -1 <br> 0 <br> 0 <br> 0 |  | $\circ$ <br> $\stackrel{\circ}{\circ}$ <br> $\stackrel{-}{\circ}$ <br> - | $\stackrel{1}{2}$ <br> $\stackrel{1}{\circ}$ <br> 0 <br> 0 | O <br> $\stackrel{3}{8}$ <br> $\stackrel{3}{\circ}$ <br> - | $\stackrel{n}{1}$ $\stackrel{3}{8}$ 0 0 | $\begin{aligned} & \text { M } \\ & 0 \\ & 0 \\ & \hline 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \hat{0} \\ & \stackrel{1}{\circ} \\ & \underset{O}{\circ} \end{aligned}$ |  | $\infty$ $\stackrel{\infty}{\infty}$ 0 0 0 0 | $\stackrel{m}{\sigma}$ <br> $\stackrel{-}{\circ}$ | $\infty$ O O or 0 0 | $\stackrel{1}{0}$ 0 0 0 0 | ㅇ g or 0 0 | $\circ$ <br> $\stackrel{9}{8}$ <br> 8 <br> - | 0 <br> 0 <br> 0 <br> 8 <br> 0 <br> 0 | 8 <br> 8 <br> 8 <br> 8 <br> - | － <br> － <br> － <br> - | $\circ$ <br> 8 <br> 8 <br> - <br> 0 |
| O | 8 0 0 0 0 0 | 0 0 0 0 0 0 0 | $\begin{aligned} & \text { O } \\ & \text { O } \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{N}{N} \\ & \underset{\sim}{\underset{O}{0}} \end{aligned}$ | $\stackrel{m}{\square}$ <br> $\stackrel{8}{8}$ <br> - | $\begin{aligned} & \circ \\ & 6 \\ & 0 \\ & \hline 0 \\ & 0 \\ & \hline 0 \end{aligned}$ | N <br>  <br> $\AA$ <br> 0 | $\begin{aligned} & 8 \\ & \stackrel{\circ}{\circ} \\ & \hline 8 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { O } \\ & \text { O} \\ & 0 \end{aligned}$ |  |  | $\circ$ <br> ö <br> O－ <br> or <br> 0 <br> 0 |  |  | $\infty$ $\stackrel{\infty}{\circ}$ $\stackrel{8}{\circ}$ 0 | $\begin{aligned} & 10 \\ & 0 \\ & \hline 8 \\ & \hline 8 \\ & \hline \end{aligned}$ | $\circ$ <br> 8 <br> - <br> - <br> - |  | $\circ$ <br> O <br> － <br> ® <br> - |
| $\stackrel{5}{8}$ | $N$ N 0 0 0 | 10 0 0 0 0 0 0 | 0 0 0 0 0 0 0 | N O N O－ |  | $N$ <br>  <br> 0 <br> 0 <br> 0 | 4 <br> 0 <br> 8 <br> - <br> - | $N$ $N$ $\stackrel{N}{\circ}$ 0 0 | $\circ$ <br> 0 <br> 8 <br> 0 <br> 0 | 0 0 0 0 0 0 | 0 <br> 8 <br> 8 <br> - <br> - |  | 0 <br> 0 <br> 8 <br> 8 <br> 0 | $\infty$ <br> 0 <br> 0 <br> 8 <br>  <br> 0 <br> 0 | $\stackrel{\infty}{\stackrel{\infty}{\circ}}$ | 10 <br> 0 <br> 0 <br>  <br> 0 <br> 0 | 8 <br> 8 <br> 8 <br> - | 앙 <br> - <br> - <br> - | 10 <br> 8 <br> 8 <br> 8 <br> 0 |
| 앙 | J N 0 0 0 | 0 0 0 0 0 0 | $\infty$ $\stackrel{\infty}{\circ}$ 0 0 0 0 | 0 0 0 0 0 0 | 0 $\stackrel{9}{2}$ 0 0 0 | $\begin{aligned} & \text { H } \\ & \text { NO } \\ & \text { O} \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 10 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | 0 <br> 0 <br> 0 <br> 0 <br> 0 | 10 0 0 0 0 0 |  | $\begin{aligned} & \text { } \\ & \text { o } \\ & \text { or } \\ & \text { ón } \end{aligned}$ | N O O 0 0 |  |  | $\begin{aligned} & \text { + } \\ & 0 \\ & \underset{\circ}{\circ} \\ & 0 \end{aligned}$ | 0 <br> 0 <br> 0 <br> 8 <br> 0 <br> 0 | 응 <br> － <br> - <br> - | 10 <br> 8 <br> 8 <br> 8 <br> 0 |
| N | $\overline{\mathrm{N}}$ | $\underset{\sim}{N}$ | $\stackrel{m}{\mathrm{~N}}$ | $\stackrel{\rightharpoonup}{\sim}$ | $\stackrel{\sim}{\sim}$ | $\stackrel{\bullet}{N}$ | $\hat{\mathrm{N}}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{n} \end{aligned}$ | $\stackrel{\circ}{\mathrm{N}}$ | $\stackrel{O}{\infty}$ | $\stackrel{\Gamma}{ल}$ | $\stackrel{N}{m}$ | $\begin{aligned} & \infty \\ & \infty \end{aligned}$ | $\stackrel{\star}{\text { m }}$ | $\stackrel{\sim}{\infty}$ | $\stackrel{\ominus}{\varphi}$ | $\hat{m}$ | $\stackrel{\infty}{\infty}$ | $\stackrel{\square}{\text { ¢ }}$ |

### 8.2 Table 2 - percentage points of the normal distribution

The table gives the values of $z$ satisfying $\mathrm{P}(Z \leq z)=p$, where $Z$ is the normally distributed random variable with mean $=0$ and variance $=1$.


| $p$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | $p$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.5 | 0.0000 | 0.0251 | 0.0502 | 0.0753 | 0.1004 | 0.1257 | 0.1510 | 0.1764 | 0.2019 | 0.2275 | 0.5 |
| 0.6 | 0.2533 | 0.2793 | 0.3055 | 0.3319 | 0.3585 | 0.3853 | 0.4125 | 0.4399 | 0.4677 | 0.4958 | 0.6 |
| 0.7 | 0.5244 | 0.5534 | 0.5828 | 0.6128 | 0.6433 | 0.6745 | 0.7063 | 0.7388 | 0.7722 | 0.8064 | 0.7 |
| 0.8 | 0.8416 | 0.8779 | 0.9154 | 0.9542 | 0.9945 | 1.0364 | 1.0803 | 1.1264 | 1.1750 | 1.2265 | 0.8 |
| 0.9 | 1.2816 | 1.3408 | 1.4051 | 1.4758 | 1.5548 | 1.6449 | 1.7507 | 1.8808 | 2.0537 | 2.3263 | 0.9 |


| $p$ | 0.000 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.008 | 0.009 | $p$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.95 | 1.6449 | 1.6546 | 1.6646 | 1.6747 | 1.6849 | 1.6954 | 1.7060 | 1.7169 | 1.7279 | 1.7392 | 0.95 |
| 0.96 | 1.7507 | 1.7624 | 1.7744 | 1.7866 | 1.7991 | 1.8119 | 1.8250 | 1.8384 | 1.8522 | 1.8663 | 0.96 |
| 0.97 | 1.8808 | 1.8957 | 1.9110 | 1.9268 | 1.9431 | 1.9600 | 1.9774 | 1.9954 | 2.0141 | 2.0335 | 0.97 |
| 0.98 | 2.0537 | 2.0749 | 2.0969 | 2.1201 | 2.1444 | 2.1701 | 2.1973 | 2.2262 | 2.2571 | 2.2904 | 0.98 |
| 0.99 | 2.3263 | 2.3656 | 2.4089 | 2.4573 | 2.5121 | 2.5758 | 2.6521 | 2.7478 | 2.8782 | 3.0902 | 0.99 |

